

BLACK HOLE ENTROPY AND STRING THEORY

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Abstract

This is an expanded version of a talk given at “*Ind Recontre du Vietnam*” held at Ho Chi Minh City in October, 1995. We discuss several aspects of black hole entropy in string theory. We first explain why the geometric entropy in two dimensional noncritical string theory is nonperturbatively finite. We then explain the philosophy of regarding massive string states as black branes and how the Beckenstein-Hawking entropy for extremal BPS black holes may be understood as coming from degeneracy of string states. This is then discussed in the context of D-strings in Type IIB superstrings. We then describe non-BPS excitations of D-strings and their entropy and explore the possibility that their decay describes Hawking radiation. For these D-strings and other D-branes the entropy and temperature are consequences of the physical motion of stuck open strings along the D-brane and this leads to a simple space-time interpretation. Finally we speculate that the horizon may be itself regarded as a D-brane.

In this contribution I will discuss some aspects of black hole entropy and how string theory has helped us to understand this rather mysterious quantity. This is based on a talk given at *Ind Recontre du Vietnam*. However I have added some developments which took place very recently to make the article meaningful at this time.

Even before the phenomenon of black hole radiation was discovered, Beckenstein [1] noticed a profound analogy between black holes in classical general relativity and the laws of thermodynamics. In particular he found that the surface gravity at the horizon of a black hole can be regarded as a temperature and the area of the horizon as an entropy. Hawking [2] showed that black holes radiate due to quantum effects with a temperature which is indeed proportional to the surface gravity and this gave a precise formula for the “Beckenstein- Hawking entropy” S_{HB} which, for simplest black holes, read

$$S_{HB} = \frac{A}{4G_N} \quad (1)$$

where A is the horizon area and G_N is Newton’s constant. In the presence of matter or higher derivative terms in the action this formula may require modification [3].

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There is a way in which the answer (1) follows from a *classical* calculation in euclidean gravity [4]. One computes the action of the euclidean black hole solution and identifies it with a free energy. Standard thermodynamics together with the formula for the temperature gives rise to the expression (1).

It has always been a big mystery why this is an entropy, since there is no obvious statistical origin of this quantity. As we will discuss later, string theory is offering a way to resolve this mystery.

1 Geometric Entropy and 2d strings

Before delving into this, let us discuss another related object which enters in black hole thermodynamics. This is “entanglement entropy” [5] [6] [7] or “geometric entropy” of black holes and measures, in a given quantum state, the correlation of the region inside the horizon with the region outside. In usual field theories of scalars and spin-1/2 particles, this is the same as the quantum correction to the Hawking-Beckenstein entropy defined through the partition function on the euclidean background with a defect angle. For gauge fields the two quantities are not the same [8]. In string theory the situation is tricky since a string can be partly inside and partly outside the horizon [6] [7]. We will come back to this aspect in a later section.

1.1 Geometric entropy

The notion of a geometric entropy exists in any theory, regardless of black holes. Consider for example some field theory in $d + 1$ dimensions. Let us divide the space into two halves by putting in an imaginary plane at $x = 0$ where x is one of the spatial coordinates. The wave functional in any given state may be then written as $\Psi[\phi_L, \phi_R]$ where ϕ_L and ϕ_R are the fields in the regions $x < 0$ and $x > 0$ respectively. If we are interested only in operators which depend on ϕ_R all observables may be expressed in terms of a reduced density matrix $\rho[\phi_R, \phi'_R]$

$$\rho[\phi_R, \phi'_R] = \int \mathcal{D}\phi_L \Psi[\phi_L, \phi_R] \Psi^*[\phi_L, \phi'_R] \quad (2)$$

One can now associate an entropy S_G with this density matrix

$$S_G = -\hat{\rho} \log \hat{\rho} \quad (3)$$

where $\hat{\rho}$ is the normalized density matrix.

In usual flat space the divide at $x = 0$ is imaginary. However for an external observer in a black hole background, the horizon provides a real division of space. Thus if we define an object like (2) with L and R being replaced by the interior and the exterior of the horizon respectively we have a physical quantity which measures the amount of information hidden inside the horizon. In a dynamical situation of collapse and subsequent evolution one may then measure the net change in the total entropy $S_{HB} + S_G$ and see whether this increases. This is indeed possible [9],[10] in CGHS-RST models of black hole evolution.

An important property of the entanglement entropy is that in field theory this quantity depends on the ultraviolet cutoff [11]. In $d + 1$ dimensions it behaves as Λ^{d-1} for $d > 1$ and $\log(\Lambda L)$ for $d = 1$ where Λ is a (momentum) ultraviolet cutoff and L is the size of the system. In a curved background one must use an invariant cutoff and as shown e.g. in [10] this means that the entanglement entropy depends on the metric at the horizon so that the time evolution of the latter determines the time evolution of the entropy. For the two dimensional models this

implies that an indefinite amount of information is lost - so long as we trust the semiclassical approximation in regions of low curvature [10].

There is a simple way to understand why the geometric entropy is logarithmically divergent in $1+1$ dimensional field theories [10]. Consider for example a free massless scalar field $\phi(x, t)$ in a spatial box of size L . We want to find the entropy of entanglement between the left and the right halves of a divide at $x = 0$. Now write the field in a basis of non-overlapping wave packets formed out of wave numbers $2^j k_0 < k < 2^{j+1} k_0$ where $k_0 = \frac{2\pi}{L}$ and j is some integer. For each value of j only one wavepacket straddles across $x = 0$ and contributes to the entropy and the contribution is independent of j due to conformal invariance. Thus the entropy is proportional to the maximum allowed value of j , $j_{max} \sim \log(L\Lambda)$. A similar analysis gives the entropy for entanglement between some region $x_1 < x < x_2$ and the rest of space. Here the relevant contribution comes from wavenumbers $k > 1/(x_2 - x_1)$ and one has

$$S_G = \log [(x_2 - x_1)\Lambda] \quad (4)$$

The entropy is infinite simply because there are an infinite number of modes which contribute to it.

In perturbative string theory there are no ultraviolet divergences, however there is now an infrared divergence associated with a hagedorn transition near the horizon [13]. The fact that the high temperature behavior of string theory enters into the discussion implies that the issue is nonperturbative.

We know very little about nonperturbative behavior of critical string theories, though it is certainly hoped that the recent developments in string duality will teach us something. We do have a nonperturbative formulation for at least one noncritical string theory - the two dimensional string defined through the double scaling limit of matrix quantum mechanics [14]. However we don't understand black holes in the matrix model formulation. Nevertheless, as we just saw, the ultraviolet divergence of the geometric entropy of field theories is present in usual flat space and one may ask the same question in this two dimensional string theory. In the following we investigate this question. Details may be found in [15].

1.2 Geometric entropy in the 2d string

The fundamental formulation of this theory is in terms of mutually noninteracting nonrelativistic fermions in an inverted harmonic oscillator potential $V(x) = -x^2/2$.

$$S = \frac{1}{2} \int dx dt \psi^\dagger(x, t) [i\partial_t + \partial_x^2 + x^2] \psi(x, t) \quad (5)$$

Let the fermi level be denoted by $-\kappa$. Then the excitations of the theory are described in terms of a massless scalar field $\xi(x, t)$ which is the fluctuation of the density of these fermions. This massless scalar field is related to the only propagating degree of freedom of two dimensional string theory - the massless "tachyon" - but not the same³. The dynamics of $\xi(x, t)$ is described in perturbation theory by the collective field hamiltonian

$$H = \int d\tau \left[\frac{1}{2} (\Pi_\xi^2 + (\partial_\tau \xi)^2) + \frac{1}{6\rho_0^2(x)} ((\partial_\tau \xi)^3 + 3\Pi_\xi (\partial_\tau \xi) \Pi_\xi) \right] \quad (6)$$

where

$$\rho_0^2(x) = x^2 - \kappa = \frac{1}{g_{eff}(x)} \quad (7)$$

³See e.g. Polchinski in [14]

is the inverse (position dependent) string coupling in the theory, and τ is a new spatial coordinate defined by $dx = \rho_0(x)d\tau$. The theory is thus strongly coupled near the hump of the potential at $x = 0$ and weakly coupled in the asymptotic region $|x| \gg \sqrt{\kappa}$. In terms of τ one has $g_{eff}^{-1}(\tau) = \kappa \sinh^2 \tau$ so that for a given τ large κ means weak coupling.

In a sense (6) is a “string field theory” whose fields are directly related to the perturbative excitations of the model. The position dependence of the string coupling is typical of noncritical strings and comes from a nontrivial dilaton background. On the other hand (5) is a more basic description of the theory.

In perturbation theory one has a massless scalar field and the lowest order answer would have the characteristic logarithmic dependence on the cutoff. Note that this is not the standard ultraviolet divergence of vacuum loop diagrams : in the collective field theory such diagrams are finite as expected in string theory. As explained in [15] this is in fact related to the high temperature behaviour of the theory. In principle one could take into account the effect of interactions using the hamiltonian (6) : this would be like computing the geometric entropy in string perturbation theory and it would be hard to imagine how this dependence on the cutoff is removed.

However here we are lucky to have a formulation of the model which is as easily analyzable in strong coupling as in weak coupling - viz. the fundamental fermionic formulation given by (5).

In fact the essential physics is present even for *free* nonrelativistic fermions in a box which we discuss first. Let k_F denote the fermi momentum so that all states in $-k_F < k < k_F$ are filled in the ground state. In the following we will define a shifted momentum $q = k - k_F$ with corresponding redefined fields (by a phase) $\chi(q) = \psi(k_F + q)$. The ground state wave functional may be written in terms of the grassmann valued fields $\chi(q)$ as

$$\Psi_0 = \exp\left[-\frac{1}{2} \int_{-\infty}^{\infty} dq \bar{\chi}(q)\chi(q) + \int_{-2k_F}^0 dq \bar{\chi}(q)\chi(q)\right] \quad (8)$$

In terms of position space fields the first term in the exponent is an integral of a *local* density. This does not entangle the left and right halves and does not contribute to the entropy. The entire contribution comes from the second term which involves only modes in the fermi sea.

This is the crucial point. Since only modes in the fermi sea contribute and the fermi sea has a finite depth, the ultraviolet cutoff does not play a role in the geometric entropy - as opposed to the case of the scalar field discussed above. However, if we expand around large k_F , in the lowest order the fermi sea of nonrelativistic fermions becomes the infinite Dirac sea. One effectively has relativistic fermions which is equivalent in two dimensions to a relativistic boson. In this limit one gets the usual logarithmically divergent answer.

The bosonized form of the theory of free nonrelativistic fermions is given by the collective field theory (6) with ρ_0 in (7) replaced by $\rho_0^2 = \mu_F = \frac{1}{2}k_F^2$. Thus expansion around $k_F = \infty$ is the perturbation expansion of the bosonic theory. We immediately see that the finiteness of the geometric entropy is an essentially *nonperturbative* phenomenon in the bosonic description.

Let us now go back to the one dimensional string, i.e. include the effect of a $-x^2$ potential. The problem we want to consider is the following : consider a box of size l centered around a point $x = -x_0$. Choose x_0 to lie in the asymptotic region where the theory is weakly coupled and l to be much smaller than the overall size of the system. We want to estimate the entanglement entropy between this box and the rest of the system. The depth of the fermi sea at x_0 is (in energy) $x_0^2 - \mu$. Thus in this calculation the relevant range of wavelengths is $\sqrt{x_0^2 - \mu} < \lambda < l$. The answer for the geometric entropy becomes [16]

$$S_G \sim \log[l^2 g_{eff}(x_0)] \quad (9)$$

where we have used (7). The answer does not depend on the ultraviolet cutoff and is clearly nonperturbative.

The above calculation drives home a point which may be quite generic in string theory : the true degrees of freedom at high energies may be rather different and fewer in number than what one expects from perturbative considerations. For critical string theory we don't know what these degrees of freedom are. For the two dimensional string the degrees of freedom are transparent since we have free nonrelativistic fermions.

2 The Beckenstein Hawking Entropy in Superstring Theory

We now turn to the question of Beckenstein-Hawking entropy. It is an old idea that very massive elementary particles behave as black holes in strong coupling[17]. Recently this idea has proved to be very fruitful in understanding the Hawking-Beckenstein entropy in string theory. The idea is as follows. It is well known that in string theory there are very massive states and there are large number of states of a given mass. Could it be that this degeneracy of states is the origin of the Beckenstein-Hawking entropy [18] [6] [12] ?

However at the face of it, the idea seems to run into trouble for usual Schwarzschild black holes. Here the entropy S for mass M is $S \sim M^2$ whereas it is known that in string theory the degeneracy of states grows as e^M . It was argued in [6], [12] that quantum effects might renormalize the mass suitably. A different proposal involving noncritical strings has been put forward in [19]

Luckily there are states in string theory whose masses are not renormalized - these are the BPS saturated states. The above idea may be tested for such states which are to be identified with extremal black holes [20]. Indeed such states do behave like extremal black holes in scattering processes [21][22][23]. However, most of these extremal holes have zero horizon area and should lead to a zero entropy ! In [24] it was, however, proposed that the area of the *stretched horizon* [7] rather than the event horizon should be identified with the entropy. Indeed for a class of such BPS saturated extremal holes in heterotic string theory compactified on T^6 it was found that the dependence of the stretched horizon area on the parameters specifying the solution agrees with the dependence of the degeneracy of string states on these parameters (which are generically charges) [24] ⁴.

2.1 D-branes

Recently this connection has been better understood owing to the realization that a certain class of solitons in superstring theories can be described in terms of objects called D-branes [26].

Consider a theory of closed strings, e.g. the Type IIB superstring theory. Now add to this some open strings whose ends are restricted to move only in p of the spatial dimensions. This means we have imposed Dirichlet boundary conditions on $(9 - p)$ coordinates. The values of these $(9 - p)$ coordinates then define a p - dimensional object moving in time - a p -brane. This p brane is a soliton in the closed string theory. The collective coordinates which describe the low energy excitations of this soliton are precisely the lowest mass modes of the open strings whose ends are stuck on the brane.

⁴ It has also been argued using thermodynamic arguments that the entropy of an extremal black hole *could* be proportional to the mass [25]

Closed superstring theories have generally two types of gauge fields coming from the NS-NS and R-R sectors on the world sheet. Consider for example the Type IIB theory. The ten dimensional gauge fields of this theory consist of (1) NS-NS sector : dilaton ϕ , metric g_{AB} and antisymmetric tensor field $B_{AB}^{(1)}$ (2) R-R sector : an axion field χ , antisymmetric tensor $B_{AB}^{(2)}$ and a rank four gauge field with self dual field strength C_{ABCD} . The object which carries charges under the NS-NS gauge field $B_{AB}^{(1)}$ is in fact the elementary string itself. At low energies the elementary string is described by a classical solution of the effective field theory - the macroscopic string [27]. However there are no states in the perturbative spectrum of the elementary string theory which carry charges of the R-R gauge field $B_{AB}^{(2)}$. Remarkably, these missing states carrying R-R charges are D-branes [28]. The field content shows that these D-branes describe p -branes with odd p for Type IIB and even p for Type IIA.

The Type-IIB theory is conjectured to be self-dual under an $SL(2, Z)$ symmetry which transform (ϕ, χ) and $(B_{AB}^{(1)}, B_{AB}^{(2)})$ into each other. In fact there are an infinite number of classical string solutions labelled by (m, n) where m denotes a quantized NS-NS charge and n a quantized R-R charge [29]. These solutions are in fact low energy descriptions of bound states of D-branes and elementary strings [30]. In particular the string with $(0, 1)$ charge has a nonzero $B^{(2)}$ (plus metric and dilaton) but zero $B^{(1)}$ - we will call this the D-string. The duality conjecture then implies that the states of the D-string behave exactly like the states of the elementary string with the coupling replaced by the inverse coupling. Evidence for this has accumulated over the past few months [30] [31].

2.2 BPS States and solutions

Consider flat ten dimensional space with one of the directions, say $X^9 = z$ compactified on a circle of circumference L . Then the mass of an elementary string state is given by

$$M^2 = (n_w LT + \frac{2\pi n_p}{L})^2 + 8\pi T N_R = (n_w LT - \frac{2\pi n_p}{L})^2 + 8\pi T N_L \quad (10)$$

where n_p is the quantized momentum in the X^9 direction and N_L, N_R denotes the oscillator number of the left and right moving oscillators on the world sheet. We have defined them so that the minimum values ($1/2$ for NS and 0 for R sectors) have been subtrated out. T is the elementary string tension. The mass formula (10) is perturbative. However there are a special class of states when this is in fact exact. These are the BPS saturated states which have *either* $N_R = 0$ with N_L arbitrary or $N_L = 0$ with N_R arbitrary. Consider the first case, $N_R = 0$. The level matching condition (10) then shows that $N_L = 4n_w n_p$.

Thus for a given (n_p, n_w) there are many states in the string theory : these correspond to the number of ways one can have a total level N_L from the basic bosonic and fermionic oscillators. As is well known the number of such states grows as $\sim e^{2\pi\sqrt{2N_L}}$ for large N_L . The low energy description of such states are oscillating macroscopic strings with string tension $\sigma = n_w T$ and a momentum density $p = 2\pi n_p / L^2$ along the string [22] [32] and carry the NS-NS charge for the gauge field $B^{(1)}$. These are in fact extremal black strings with the horizon coinciding with the singularity. If five of the transverse dimensions as well as the string direction is compactified they appear as charged black holes in four dimensions carrying two kinds of $U(1)$ charge : one coming from the $B^{(1)}$ charge and one from the $g_{z\mu}$ component of the metric. This is in fact one of the solutions considered in [24] though in that case the origin of the charges is rather different.

The classical solution representing this black string (or black hole in the compactified theory) is specified by only the string tension and the charge density. However identification of this

with a BPS string state shows that there are many possible states with the same set of charges, corresponding to the many ways of making N_L from the basic string oscillators. It is therefore, logical to assign an *entropy* to such a black string which is the logarithm of the degeneracy. For large charges

$$S = 2\pi\sqrt{8n_w n_p} = L\sqrt{2\pi\sigma p} \quad (11)$$

As mentioned earlier this agrees with the area of the stretched horizon.

We now describe the excitations for the D-string carrying RR charge rather than NS-NS charge [33]. The classical solutions may be obtained by the duality transformation described above. The solution is the same as the oscillating string NS-NS solution with the following changes : (1) the elementary string tension T is replaced by the D-string tension $T_D = T/g$ where g is the string coupling (2) $B^{(1)}$ is replaced by $B^{(2)}$ and (3) the dilaton ϕ is replaced by $-\phi$. The description of the underlying string states is however rather different and leads to a rather interesting picture for the entropy.

The low lying states of the D-string are described by the lowest modes of the Dirichlet open string theory. The physical single string states of lowest (zero) mass are given by eight transverse vectors and their supersymmetric partners. These can move only along the D-string due to Dirichlet boundary conditions and being massless can be either left or right moving along the D-string. Now consider many such open strings moving along the D-string. Then a collection of such strings all moving in the same direction (i.e. all left or all right handed) constitutes a BPS saturated state with some net momentum along the string. These are the dual analogs of the NS-NS charged states described by (10). Note that the length of a D-string wound n_w times around the X^9 direction is $n_w L$ so that the momenta of the individual open string states can be $\frac{2\pi m}{n_w L}$ for integer m . However we require the *total* momentum of the state to be $\frac{2\pi n_p}{L}$. We thus have

$$n_p = \sum_i \sum_m n_m^{(i)} \frac{m}{n_w} \quad (12)$$

where $n_m^{(i)}$ denotes the number of single open string states with momentum $\frac{2\pi m}{n_w L}$ and (i) is the label for the vector or spinor index of the state. We immediately see that the counting is exactly the same as for the NS-NS states since there are eight transverse directions for the D-string. This is of course expected from duality. The crucial difference between the elementary string and the D-string descriptions is that in the former a large number of excitations of *single string* modes was responsible for the entropy. For the D-string, however, the excitations are *multiple string* states of the Dirichlet open strings.

The BPS states described above correspond to extremal black holes with vanishing horizon. It was proposed in [35] that the same philosophy should be valid for extremal holes with a nonzero horizon area [36]. Recently string theoretic understandings of such solutions have been achieved in terms of D-branes [37] [38] and the result is that the counting problem leads to precisely to the formula (1).

2.3 Non-BPS states : Hawking radiation ?

The BPS black holes and black strings described above are stable or marginally stable objects and do not radiate. The interesting objects to consider are non-BPS states. Such a state is unstable and will decay to a stable BPS extremal state, and the real question is : does this decay resemble Hawking radiation ?

Let us discuss these non-BPS states in the D-string spectrum [33]. For large L the relevant excitations are again multiple open string states but now the stuck strings can move in either

direction along the D-string. We may excite a non-BPS state starting with a BPS state characterized by n_p by adding pairs of stuck open strings moving in opposite directions so that the total momentum is still $2\pi n_p/L$. If we add n such pairs the entropy obtained from counting is

$$S_{non-BPS} = 2\pi[\sqrt{2(|n_p| + n)n_w} + \sqrt{2nn_w}] \quad (13)$$

In perturbation theory the mass of such a state is given by

$$M = M_{BPS} + \frac{4\pi n}{L} \quad (14)$$

which agrees with the mass of corresponding objects in the elementary string spectrum upto terms of order $O(1/TL^3)$. Furthermore the inelastic threshold for excitation of such a state, $\frac{4\pi}{L}$ agrees with the corresponding threshold for excitation of a NS-NS charged state as calculated in [23]. For finite L this formula is valid for weak coupling, but for large L corrections to the mass formula are suppressed by terms of order $O(g/Tn^2L^3)$ and one can trust the masses and the counting for moderately strong coupling as well. A pair of such open stuck strings may annihilate and decay into a closed string which may escape out to infinity. Note that the decay rate is also suppressed by inverse powers of L [33] [34]. Thus for large L the states are almost stable and the counting of states makes sense. Non-BPS states have been also discussed in [38] and [39] for cases where the extremal solution has nonzero horizon area and recently in [41] for extremal solutions which have a vanishing horizon area.

The above scenario for black string entropy makes sense if the decay of such a non-BPS state resembles Hawking radiation. The reason why it might be like radiation is that there are a large number of states for a given value of the macroscopic parameters (mass and charge) and one should sum over the initial states. This same degeneracy of states is responsible for the entropy (13). Thus outgoing particles would have a thermal distribution with a temperature $T = (\frac{\partial S}{\partial E})^{-1}$ where E is the energy of the outgoing particle. Noting that $E = \frac{4\pi n}{L}$ in this decay process and using (13) one gets a “temperature” for $n \ll n_p$

$$T = \frac{2}{L}\sqrt{2n} \quad (15)$$

Is this the Hawking temperature? A slightly different but equivalent calculation in [38] for decay of non-BPS excitations above BPS black holes with nonzero horizon area (obtained by compactifying the 1-brane together with another 5-brane) showed that there is an exact agreement between the temperature calculated in this fashion and the Hawking temperature of the non-extremal black hole. However, this calculation is valid for weak coupling where the horizon is actually smaller than the string scale (see also [39]). We believe that the large L limit may be more tractable for reasons given above.

2.4 Space-time interpretation

For black strings carrying NS-NS charges the Beckenstein Hawking entropy came from the large number of oscillator degrees of freedom of the string state. So far as the macroscopic object is concerned these are “internal” degrees of freedom.

For D-strings however something remarkable has happened. The entropy comes from the physical motion of the lowest modes of the stuck open strings along the D-string. These are in fact momentum modes of a *field theory* of massless particles (eight scalars and eight fermions) on the D-brane worldvolume - the supersymmetric gauge theory discussed in [30]. These degrees of freedom on the brane volume give rise to the microstates necessary for the thermodynamic behavior of the black string (or hole).

What he have here is almost normal thermodynamics of massless particles moving on the two dimensional worldsheet of the D-string. However since there is a net momentum along the D-string the right and left moving modes have different total energies. The left movers have a total energy of $[2\pi(n_p + n)/L]$ while the right movers have energy $[2\pi n/L]$. For d massless scalars and d massless fermions standard thermodynamics predicts that the entropy is

$$s = \sqrt{\frac{\pi d L E}{2}} \quad (16)$$

for left and right movers separately. With the appropriate expression for the energies and $d = 8$ for the eight transverse directions this reduces to the two terms in (13). These particles have a “temperature” which is the temperature of the equivalent canonical ensemble, this is $\theta = \sqrt{\frac{8E}{\pi d L}}$, i.e.

$$\begin{aligned} \theta &= \frac{1}{L} \sqrt{2n} && \text{for right movers} \\ \theta &= \frac{1}{L} \sqrt{2(|n_p| + n)} && \text{for left movers} \end{aligned} \quad (17)$$

Note that in defining these temperatures we have kept the charges n_p and n_w fixed. Furthermore these temperatures are not the same as the candidate Hawking temperature in (15). A similar temperature of left movers have been defined in [38], but differ by a factor of two from the above.

The D-string description thus gives a useful physical picture of black hole thermodynamics. There are these open strings stuck to the D-string. The outside observer in the asymptotic region cannot see the details of the motion of these strings along the brane and therefore decides to average over them : this gives rise to thermodynamics in a way entirely similar to the way thermodynamics appears in ordinary processes. It is likely that this rather simple physical picture will be useful to study the black hole problem.

Note that for the D-string the target space fields of the open string become coordinates on the D-string worldsheet. Duality of the IIB theory ensures that the oscillator count of the elementary string is the same as the count of open string momentum modes. However for D-branes of higher dimensionality the open string massless fields are coordinates on a *higher* dimensional worldvolume. Nevertheless the thermodynamics is that of some physical particles moving on this worldvolume ⁵.

3 The Horizon as a D-brane

In this section we briefly mention some speculative ideas about entanglement entropy in string theory and its relation to D-branes [40]. As discussed in [6] and [7] in string theory it is tricky to define an entanglement entropy since a string can be partly inside and partly outside the horizon. To an outside observer using coordinates which stop at the horizon, such strings would actually appear as *open strings* whose ends move along the horizon. But this means that the horizon behaves like a D-brane !

To make this idea a little more concrete we can imagine working in a fixed gauge. Consider closed string theory in flat space with an imaginary divide at $x = 0$, like in Section 1. The wave functional of a string state may be schematically written as

$$\Psi[\Phi(X_L(\sigma)), \Phi(X_R(\sigma)), \Phi(X_H(\sigma)); t] \quad (18)$$

⁵For an example on 3-branes see [41]

where $X_L(X_R)$ denote strings which are entirely in $x < 0$ ($x > 0$) and X_H denote strings which are half inside and half outside and Φ the corresponding string fields. Roughly speaking the string field Φ_H may be expressed as a product of open string fields living on the left and the right. Then we may proceed to define an entanglement entropy by integrating over the closed string fields on the left as well as the open string fields on the left. The density matrix is now functionals of a closed string field Φ_R and an open string field $\Phi_{O,R}$ and the latter corresponds to open strings which are stuck on the horizon. A fixed area path integral would now involve both closed and open string diagrams. For a black hole one would have the euclidean black hole background with a conical singularity. It is likely that such quantities may be obtained by using D-brane technology. We note while the above picture is related to (and inspired by) [6] [7] there may be some differences.

In fact the above picture immediately leads to a finite contribution to the black hole entropy proportional to the horizon area from the open string states stuck to the horizon. This is because the effective action for the modes of such strings is in fact a Nambu-Goto action on the D-brane worldsheet, coming from the disc diagram. It is important to note that we are doing an euclidean calculation so that, e.g. in four dimensions, the space has a topology of $S^2 \times R^2$ and the horizon is actually the S^2 at the origin of R^2 (and not $S^2 \times R$).

This picture of viewing the horizon itself as a D-brane could give a realization of the membrane paradigm in string theory. Furthermore this picture looks tantalizingly similar to the more concrete picture of black string thermodynamics in the previous section if the D-string is actually the horizon rather than the singularity. In fact in [38] it was found that the classical geometry for the 1-brane and 5-brane configurations seem to show that these branes are actually located at the horizon. While we do not fully understand how and why this happens, it is possible that this fact will help us to concretize the picture of the horizon as a D-brane.

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